19.

$$\int \sin(3x)\cos(5x)dx = \sin(3x)\cdot\frac{1}{5}\sin(5x) - \frac{3}{5}\int \sin(5x)\cos(3x)dx = \frac{1}{5}\sin(3x)\sin(5x) - \frac{3}{5}\int \sin(5x)\cos(3x)dx =$$

$$u = \sin(3x) \Rightarrow du = 3\cos(3x)dx$$

$$dv = \cos(5x)dx \Rightarrow v = \int \cos(5x)dx = \frac{1}{5}\sin(5x)$$

$$\frac{1}{5}\sin(3x)\sin(5x) - \frac{3}{5}\left[\cos(3x)\cdot -\frac{1}{5}\cos(5x) - \int -\frac{1}{5}\cos(5x)\cdot -3\sin(3x)dx\right] =$$

$$u = \cos(3x) \Rightarrow du = -3\sin(3x)dx$$

$$dv = \sin(5x)dx \Rightarrow v = \int \sin(5x)dx = -\frac{1}{5}\cos(5x)$$

$$\frac{1}{5}\sin(3x)\sin(5x) + \frac{3}{25}\cos(3x)\cos(5x) - \frac{9}{25}\int \sin(3x)\cos(5x)dx$$

A loop exists - original question found in caluclated answer $\therefore$

$$\int \sin(3x)\cos(5x)dx = \frac{1}{5}\sin(3x)\sin(5x) + \frac{3}{25}\cos(3x)\cos(5x) - \frac{9}{25}\int \sin(3x)\cos(5x)dx$$

$$\frac{34}{25}\int \sin(3x)\cos(5x)dx = \frac{1}{5}\sin(3x)\sin(5x) + \frac{3}{25}\cos(3x)\cos(5x)$$

$$\int \sin(3x)\cos(5x)dx = \frac{25}{34}\left(\frac{1}{5}\sin(3x)\sin(5x) + \frac{3}{25}\cos(3x)\cos(5x)\right)$$

20.

$$\int \cos x \ln(\sin x)dx = \sin x \ln(\sin x) - \int \sin x \cdot \frac{1}{\sin x}\cdot \cos x dx = \sin x \ln(\sin x) - \int \cos x dx =$$

$$u = \ln(\sin x) \Rightarrow du = \frac{1}{\sin x}\cdot \cos x dx$$

$$dv = \cos x dx \Rightarrow v = \int \cos x dx = \sin x$$

$$\sin x \ln(\sin x) - \sin x = \sin x\left(\ln(\sin x) - 1\right)$$

21.

$$\int (2x+3)e^x dx = (2x+3)e^x - \int e^x \cdot 2dx = (2x+3)e^x - 2\int e^x dx = (2x+3)e^x - 2e^x =$$

$$u = (2x+3) \Rightarrow du = 2dx$$

$$dv = e^x dx \Rightarrow v = \int e^x dx = e^x$$

$$e^x\left((2x+3) - 2\right) = e^x\left(2x+1\right)$$

$$\int x5^x\,dx = x\cdot\frac{1}{\ln 5}\cdot 5^x - \int \frac{1}{\ln 5}\cdot 5^x dx = \frac{1}{\ln 5}x\cdot 5^x - \frac{1}{\ln 5}\int 5^x\,dx = \frac{1}{\ln 5}x\cdot 5^x - \frac{1}{\ln 5}\cdot\frac{1}{\ln 5}\cdot 5^x =$$

$$u = x \Rightarrow du = dx$$

22. $$dv = 5^x\,dx \Rightarrow v = \int 5^x\,dx = \frac{1}{\ln 5}\cdot 5^x$$

$$\frac{1}{\ln 5}5^x\left(x - \frac{1}{\ln 5}\right)$$

$$\int x^5 e^{x^2}\,dx = \int x^4\cdot x e^{x^2}\,dx = x^4\cdot\frac{1}{2}e^{x^2} - \int \frac{1}{2}e^{x^2}\cdot 4x^3 dx = \frac{1}{2}x^4 e^{x^2} - 2\int x^2\cdot x\cdot e^{x^2}\,dx =$$

$$u = x^4 \Rightarrow du = 4x^3 dx$$

$$dv = x e^{x^2}\,dx \Rightarrow v = \int x e^{x^2}\,dx = \frac{1}{2}\int e^z dz = \frac{1}{2}e^z = \frac{1}{2}e^{x^2}$$

$$z = x^2 \Rightarrow dz = 2xdx \Rightarrow \frac{1}{2}dz = xdx$$

23. $$\frac{1}{2}x^4 e^{x^2} - 2\left[x^2\cdot\frac{1}{2}e^{x^2} - \int\frac{1}{2}e^{x^2}\cdot 2xdx\right] = \frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + 2\int x e^{x^2}\,dx =$$

$$u = x^2 \Rightarrow du = 2xdx$$

$$dv = x e^{x^2}\,dx \Rightarrow v = \frac{1}{2}e^{x^2} \quad \text{solved above.}$$

$$\frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + 2\cdot\frac{1}{2}e^{x^2} = \frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} = e^{x^2}\left(\frac{1}{2}x^4 - x^2 + 1\right)$$

Integral component solved using the "*dv*" from the first steps

$$\int (\ln x)^2\,dx = x(\ln x)^2 - \int x\cdot 2\ln x\cdot\frac{1}{x}dx = x(\ln x)^2 - 2\int \ln x dx =$$

24. $$u = (\ln x)^2 \Rightarrow du = 2\ln x\cdot\frac{1}{x}dx$$

$$dv = dx \Rightarrow v = \int dx = x$$

$$(\ln x)^2 - 2\left(x\ln x - x\right) = (\ln x)^2 - 2x\ln x - 2x$$

25.

$$\int x^3 \ln x dx = \frac{x^4}{4}\cdot \ln x - \int \frac{x^4}{4}\cdot\frac{1}{x}dx = \frac{x^4}{4}\cdot\ln x - \frac{1}{4}\int x^3 dx = \frac{1}{4}x^4\ln x - \frac{1}{4}\cdot\frac{x^4}{4} =$$

$$u = \ln x \Rightarrow du = \frac{1}{x}dx$$

$$dv = x^3 dx \Rightarrow v = \int x^3 dx = \frac{x^4}{4}$$

$$\frac{1}{4}x^4\ln x - \frac{1}{16}x^4 = \frac{1}{4}x^4\left(\ln x - \frac{1}{4}\right)$$

26.

$$\int x^3\sqrt{9-x^2}dx = \int x^2\cdot x(9-x^2)^{\frac{1}{2}}dx = x^2\cdot\frac{1}{3}(9-x^2)^{\frac{3}{2}} - \int\frac{1}{3}(9-x^2)^{\frac{3}{2}}\cdot 2xdx = \frac{1}{3}x^2(9-x^2)^{\frac{3}{2}} - \frac{2}{3}\int x(9-x^2)^{\frac{3}{2}}dx =$$

$$u = x^2 \Rightarrow du = 2xdx$$

$$dv = x(9-x^2)^{\frac{1}{2}}dx \Rightarrow v = \int x(9-x^2)^{\frac{1}{2}}dx = \frac{1}{2}\int z^{\frac{1}{2}}dz = \frac{1}{2}\cdot\frac{z^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{3}z^{\frac{3}{2}} = \frac{1}{3}(9-x^2)^{\frac{3}{2}}$$

$$z = (9-x^2) \Rightarrow dz = 2xdx \Rightarrow \frac{1}{2}dz = xdx$$

$$\frac{1}{3}x^2(9-x^2)^{\frac{3}{2}} - \frac{2}{3}\cdot\frac{1}{5}(9-x^2)^{\frac{5}{2}} = \frac{1}{3}x^2(9-x^2)^{\frac{3}{2}} - \frac{2}{15}(9-x^2)^{\frac{5}{2}} = \frac{1}{3}(9-x^2)^{\frac{3}{2}}\left(x^2 - \frac{2}{5}(9-x^2)\right)$$

$$\int x(9-x^2)^{\frac{3}{2}}dx = \frac{1}{2}\int z^{\frac{3}{2}}dz = \frac{1}{2}\cdot\frac{z^{\frac{5}{2}}}{\frac{5}{2}} = \frac{1}{5}z^{\frac{5}{2}} = \frac{1}{5}(9-x^2)^{\frac{5}{2}}$$

$$z = (9-x^2) \Rightarrow dz = 2xdx \Rightarrow \frac{1}{2}dz = xdx$$